Module 8:
Shear and Diagonal Tension in Beams

Shear and Diagonal Tension
Shear and Flexural Stress

Shear stress: \( v = \frac{VQ}{Ib} \)

Flexural stress: \( f = \frac{My}{I} \)

Principal stress per Mohr’s circle

\( t = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + v^2} \)

tension

compression
Plain Concrete Beam

\[ v = \frac{VQ}{Ib} \]
\[ f = \frac{My}{I} \]

Principal Stresses

\[ t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \]

Figure 4.3: Stress trajectories in plain concrete beam.
Diagonal Tension Cracking

\[ v_{cr} = \frac{V_{cr}}{bd} = 3.5 \sqrt{f'_c} \]

Figure 4.5. Diagonal tension cracking in reinforced concrete beams.

Concrete Shear Strength

\[ v_{cr} = \frac{V_{cr}}{bd} = 1.9 \sqrt{f'_c} + 2500 \left( \frac{\rho Vd}{M \sqrt{f'_c}} \right) \leq 3.5 \sqrt{f'_c} \]

\[ v_{cr} = \frac{V_{cr}}{bd} = 2.0 \sqrt{f'_c} \]

Figure 4.6. Correlation of Eq 4.3a with test results.
Internal Shear Forces

Figure 4.9

\[ V_{int} = V_{cz} + V_d + V_{iy} + V_s \]

\[ V_{int} = V_c + V_s \]

Figure 4.10. Redistribution of internal shear forces.

Shear Design

\[ V_u = \phi(V_c + V_s) \quad \phi = 0.75 \text{ for shear} \]

\[ \frac{V_c}{bd} = 1.9\sqrt{f'_c} + 2500 \frac{\rho V_d}{M\sqrt{f'_c}} \leq 3.5\sqrt{f'_c} \]

\[ V_s = A_v f_y \frac{d}{s} \]

\[ s_{\text{max}} = \frac{d}{2} \]

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Shear Demand, $V_u$

Figure 4.12: Locations of critical design shear

![Diagram showing shear demand at end supported beams and beam-column framing.]

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Shear Demand, $V_u$

Figure 4.12: Locations of critical design shear

![Diagram showing shear demand under concentrated load and load near bottom of beam.]

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Shear Demand, $V_u$

Figure 4.12: Locations of critical design shear

Minimum Web Reinforcement

For regions of light shear force:

- if $V_u \leq \phi V_c$ then no $A_v$ required, but....

$$A_{v,\min} = 0.75 \sqrt{f' c} \frac{b_v s}{f_y} \geq 50 \frac{b_v s}{f_y}$$

- if $V_u \leq \frac{\phi V_c}{2}$ then no $A_v$ required
Example

Design stirrups at “d” out from support:

\[ \phi V_s = V_u - \phi V_c = 76.8^k - 33.4^k = 43.4^k \]

Minimum stirrups:

\[ \frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{43.4^k}{60^{\text{kpsi}} (22^\text{in})} = 0.0438^\text{in} \]

\[ \frac{0.75}{s_{\text{min}}} = \frac{A_v}{50 b_u} = \frac{0.22(60,000)}{50(16^\text{in})} = 16.5^\text{in} \]

\[ s_{\text{min}} = \frac{d}{2} = 11^\text{in} \]

\[ s = \frac{0.11^\text{in} \times 2 \text{ legs}}{0.0438^\text{in}} = 5.0^\text{in} \]
Example

\[
\frac{V}{bd} = 1.9 \sqrt{f'_c} + 2500 \frac{\rho V d}{M \sqrt{f'_c}} \leq 3.5 \sqrt{f'_c}
\]

note \( \frac{M}{V} \) varies along span

\[\phi V_s = V_u - \phi V_c = 43.4^k\]

\[
\frac{A_s}{s} = \frac{V_s}{f_y d}
\]

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